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Increasing life expectancy and self-stabilizing pension systems. A comparison between the Swedish and the Austrian model

Markus Knell

Oesterreichische Nationalbank

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*The content of these slides reflects the views of the authors and not necessarily those of the OeNB.

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Motivation

- Pension systems have to cope with two two demographic developments:
 - Increases in life expectancy.
 - Fluctuations (and mostly declines) in fertility
- I deal with the first aspect, since it represents an *ongoing* process with considerable and far-reaching budgetary consequences.

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Automatic Adjustment Rules

- "Around half of OECD countries have elements in their mandatory retirement-income provision that provide an automatic link between pensions and a change in life expectancy [...] The rapid spread of such life-expectancy adjustments has a strong claim to be the most important innovation of pension policy in recent years" (OECD, *Pensions at a Glance*, 2011, p. 82).
- Despite this claim there does not exist much research on this "most important innovation".

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Content and Main Findings

• I focus on two pension systems:

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- The (Swedish) notional defined contribution (NDC) system
- The (Austrian) defined benefit system APG
- The NDC system can be constructed such that is has an self-stabilizing budget even if life expectancy increases in a linear fashion. For this to be the case one has to set two crucial parameters (the "notional interest rate" and the "conversion factor") in an appropriate way.
- The APG system is currently not designed in a way that it reacts automatically to changes in life expectancy. It *could* be adapted to do so, but this would involve rather complicated and less transparent adjustments.

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Notation 1

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The generation born in period t has:

- cohort size N(t) = N
- life expectancy T(t)
- retirement age R(t)

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Notation 1

The generation born in period t has:

- cohort size N(t) = N
- life expectancy T(t)
- retirement age R(t)

NOTE 1: I assume that all members of one generation reach the cohort-specific maximum age T(t).

NOTE 2: The maximum age *observed* in period t is denoted by $\tilde{T}(t)$ and the retirement age by $\tilde{R}(t)$. In general: $T(t) \neq \tilde{T}(t)$ and $R(t) \neq \tilde{R}(t)$.

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Notation 2

For generation t the PAYG system stipulates the following income streams:

• Contributions: $\tau(t+a)W(t+a)$ for $0 \le a < R(t)$ • Pensions: P(t,a) for R(t) < a < T(t)

NOTE: In NDC systems the contribution rate is fixed, i.e. $\tau(t) = \hat{\tau}$.

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Budget of the PAYG system

For the system in period *t*:

- Labor force: $L(t) = \widetilde{R}(t) \times N$
- Retired population: $B(t) = \left(\widetilde{T}(t) \widetilde{R}(t)\right) \times N$
- Average pension: $\overline{P}(t) = \frac{\int_{\widetilde{R}(t)}^{\widetilde{T}(t)} P(t-a,a) \, \mathrm{d}a}{\widetilde{T}(t) \widetilde{R}(t)}$
- Dependency ratio: $z(t) = \frac{B(t)}{L(t)} = \frac{\widetilde{T} \widetilde{R}(t)}{\widetilde{R}(t)}$
- The balanced budget condition is given by:

$$\underbrace{\tau(t)W(t)L(t)}_{\text{Revenue}=I(t)} = \underbrace{\overline{P}(t)B(t)}_{\text{Expenditure}=O(t)}$$

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The development of life expectancy

An old controvery—How to best model life expectancy?

• Life expectancy increases in a linear fashion • Graph :

 $T(t) = T(0) + \gamma \cdot t$

- Robust relationsship: In the data: γ between 0.15 and 0.33.
- From $T(t \widetilde{T}(t)) = \widetilde{T}(t)$ it follows that: $\widetilde{T}(t) = \frac{1}{1+\gamma}T(t)$.

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NDC systems

Basic features:

- Fixed contribution rate: $au(t) = \widehat{ au}$
- Life-time assessment period
- Past contributions are revalued with an appropriate notional interest rate
- At retirement the notional capital is transformed into annual pension payments by taking the development of life expectancy into account
- Advantages: Close relation between contributions and benefits; flexibility in retirement age with automatic reaction of the pension level to the age of retirement; individual accounts and annual statements increase transparency; transnational portability.



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Why focus on NDC?

- It is increasingly popular (Sweden and around 10 other countries).
- The World Bank, OECD and European Commission often use it as a reference points or benchmark to discuss reforms.
- They are explicitely designed to deal with increasing life expectancy.
- Other systems (German earnings-point, Austrian APG) can be directly related to it.

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A formal expression of NDC Systems 1

• The notional capital before retirement:

$$K(t,R(t)) = \int_0^{R(t)} \widehat{\tau} W(t+a) e^{\int_{t+a}^{t+R(t)} \rho(s) \, \mathrm{d}s} \, \mathrm{d}a,$$

where $\rho(s)$ stands for the notional interest rate in period s.

• The first pension payment:

$$P(t,R(t)) = rac{K(t,R(t))}{\Gamma(t,R(t))},$$

where $\Gamma(t, R(t))$ is the remaining life expectancy of generation t at age R(t).

• Existing pensions are adjusted according to:

$$P(t,a) = P(t,R(t))e^{\int_{t+R(t)}^{t+a}\vartheta(s)\,\mathrm{d}s},$$

where $\vartheta(s)$ stands for the adjustment rate in period s.

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A formal expression of NDC Systems 2

$$O(t) = \widehat{\tau}N \int_{\widetilde{R}(t)}^{\widetilde{T}(t)} \frac{\int_{0}^{R(t-a)} \left[W(t-a+b) e^{\int_{t-a+b}^{t-a+R(t-a)} \rho(s) \, \mathrm{d}s} \right] \, \mathrm{d}b}{\Gamma(t-a, R(t-a))} e^{\int_{t-a+R(t-a)}^{t} \vartheta(s) \, \mathrm{d}s} \, \mathrm{d}a$$

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A formal expression of NDC Systems 2

$$O(t) = \widehat{\tau} N \int_{\widetilde{R}(t)}^{\widetilde{\tau}(t)} \frac{\int_{0}^{R(t-a)} \left[W(t-a+b) e^{\int_{t-a+b}^{t-a+R(t-a)} \rho(s) \, \mathrm{d}s} \right] \, \mathrm{d}b}{\Gamma(t-a, R(t-a))} e^{\int_{t-a+R(t-a)}^{t} \vartheta(s) \, \mathrm{d}s} \, \mathrm{d}a$$

- Crucial task for the policymaker: Determine the control variables $\rho(t)$, $\vartheta(t)$ and $\Gamma(t, R(t))$ in such a way that expenditures develop in line with revenues $I(t) = \hat{\tau}L(t)W(t)$.
- Question: Is this possible for any path of the retirement age R(t) (which is the choice variable of the households)?

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The first important parameter in NDC systems — The notional interest rate

Growth rate of average wages:

$$ho(t) = g^W(t) = rac{\dot{W}(t)}{W(t)}$$

• Growth rate of the wage bill:

$$\rho(t) = g^{W}(t) + g^{L}(t) = \frac{\dot{W}(t)}{W(t)} + \frac{\dot{L}(t)}{L(t)}$$

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$$\rho(t) = g^{W}(t) + g^{L}(t) = \frac{\dot{W}(t)}{W(t)} + \frac{\dot{L}(t)}{L(t)}$$

Conventional wisdom: Use the growth rate of the wage bill.

"Viewed from a macroeconomic perspective, the 'natural' rate of return for an NDC system is the implicit return of a PAYG system: that is, the growth rate of the contribution bill" (Börsch-Supan, 2003)

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An important caveat

• If the retirement age increases, then the labor force grows – even if the cohort size is constant.

$$L(t) = \widetilde{R}(t)N o g^{L}(t) = rac{\widetilde{R}(t)}{\widetilde{R}(t)}$$

• Increases in the retirement age are, however, necessary to stabilize the dependency ratio z(t). In particular:

$$z(t) = \frac{\widetilde{T}(t) - \widetilde{R}(t)}{\widetilde{R}(t)} = \widehat{z}$$
 implies that:

$$\widetilde{R}(t) = rac{\widetilde{T}(t)}{1+\widehat{z}} = rac{T(t)}{(1+\gamma)(1+\widehat{z})}$$

• In this case: $g^{L}(t) = \frac{\gamma}{T(t)}$

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• In this case: $g^L(t) = \frac{\gamma}{T(t)}$

 \rightarrow A third concept for the notional interest rate

• "Life-expectancy adjusted" growth rate of the wage bill: $\rho(t) = \frac{\dot{W}(t)}{W(t)} + \frac{\dot{L}(t)}{L(t)} - \frac{\gamma}{T(t)}$ Example: $\gamma = 0.2$, $T(t)=60 \rightarrow \frac{\gamma}{T(t)} = 0.33\%$

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The second important parameter in NDC systems — Remaining life expectancy

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• Period (cross-section) life expectancy:

$$\Gamma(t, R(t)) = \widetilde{T}(t + R(t)) - R(t)$$

• Cohort (forecasted) life expectancy:

$$\Gamma(t, R(t)) = T(t) - R(t)$$

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The second important parameter in NDC systems — Remaining life expectancy

• Period (cross-section) life expectancy:

$$\Gamma(t, R(t)) = \widetilde{T}(t + R(t)) - R(t)$$

• Cohort (forecasted) life expectancy:

$$\Gamma(t, R(t)) = T(t) - R(t)$$

Conventional wisdom: Use cohort life expectancy

"The generic NDC annuity embodies [...] cohort life expectancy at the time the annuity is claimed" (Palmer, 2006).

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Benchmark result — a self-stabilizing budget

Assumptions:

- Constant cohort size
- Linearly increasing life expectancy
- Retirement age proportional to life expectancy: $R(t) = \mu T(t).$

Result:

- A NDC system leads to a balanced budget if the following two conditions are fulfilled:
 - The notional interest rate is equal to the adjusted growth rate of the wage bill
 - The annuity is calculated by using period life expectancy.

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R(t) proportional to T(t)

For $R(t) = \mu T(t)$ the deficit-ratio $d(t) = \frac{O(t)}{I(t)}$ is given by:

	(1)	(2)
	Notional In	terest Rate — Growth Rate of:
	Wage Bill	Adjusted Wage Bill
Period Life Expectancy	$pprox 1+rac{\gamma}{2}$	1
Cohort Life Expectancy	$pprox 1 - rac{\gamma}{2}$	$rac{1}{1+\gamma}$

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Extensions for various other assumptions about retirement behavior

- R(t) is constant \rightarrow Almost balanced.
- R(t) is optimally chosen → (for specific assumptions) balanced or almost balanced.
- R(t) is random \rightarrow Balanced over time.

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R(t) is constant

For $R(t) = \overline{R}$ the deficit-ratio $d(t) = \frac{O(t)}{I(t)}$ is given by:

	(1) (2)			
	Notional In	terest Rate — Growth Rate of:		
	Wage Bill	Adjusted Wage Bill		
Period Life Expectancy	$\approx 1 + \frac{\gamma}{2}$	≈ 1		
Cohort Life Expectancy	$pprox 1 - rac{\gamma}{2}$	$pprox 1 - \gamma$		

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R(t) is optimally chosen — Case 1

$$\mathbb{U} = \int_0^{T(t)} e^{-\delta a} U(C(t,a)) \,\mathrm{d}a - \int_0^{R(t)} e^{-\delta a} V(T(t),a) \,\mathrm{d}a,$$

where V(T(t), a) captures the disutility of work of generation t at age a.

Assume: $r = g = \delta = 0$. Case 1:

- V(T(t), a) is homogeneous of degree 0.
- $R^*(t) = \mu T(t) \rightarrow \text{Budget is always balanced.}$

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R(t) is optimally chosen — Case 2

Case 2:

- V(T(t), a) = va.
- $R^*(t) = \sqrt{\frac{T(t)}{v}} \rightarrow \text{Budget is almost balanced.}$

	(1)	(2)
	Notional In	terest Rate — Growth Rate of:
	Wage Bill	Adjusted Wage Bill
Period Life Expectancy	$pprox 1+rac{\gamma}{2}$	≈ 1
Cohort Life Expectancy	$pprox 1 - rac{\gamma}{2}$	$pprox 1 - \gamma$

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• Retirement age is a random variable. In particular:

 $R(t) = \text{Uniform} \left(0.53 \times T(t), 0.89 \times T(t)\right).$

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The Austrian APG — A Defined Benefit Account System 1

- The main element of the new harmonized Austrian pension system (APG) is an individual defined benefit account.
- The target of the system is expressed by the formula 45/65/80: After 45 years of insurance and with a retirement age of 65 the pension will amount to 80% of average lifetime income.
- This target is implemented by specifying an "accrual rate" that determines the pension claim that is associated with each income stream and is credited to the account. In Austria it is given by 1.78% (note that $45 \times 1.78 = 80$). The account is revalued with the growth rate of the average contribution basis.
- For early or late retirement within the pension corridor between 62 and 68 there are annual deductions or supplements of 5.1% (originally 4.2%).

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The Austrian APG — A Defined Benefit Account System 2

- Contribution rate: 22.8% (employees: 10.25%, employers: 12.55%).
- Ongoing pension are adjusted with the inflation rate.
- For non-contributory periods there will be tax-financed credits to the pension account.
- For the transition to the new system the original plan was to implement a "'parallel calculation". This, however, will now be substituted by an "initial pension account credit" that will be granted in 2014.

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Similiarities between the two systems

The Swedish and the Austrian system share a number of properties:

- There is a lifelong assessment period and each year of contributions has the same weight.
- Past contributions are "revalued" in an adequate manner based on the development of earnings.
- There are deductions and supplements for early and late retirement.
- Non-contributory periods are taken into account and are if possible adequately financed.
- There exists a minimum pension.
- Both systems use a transparent accounts and provide (annual) statements.

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Difference between the two models

There exists one main difference:

• The Swedish models reacts in an automatic manner to demographic changes (in particular to an increase in life expectancy). In the Austrian system there does not exist such an automatic mechanism.

In the following I want to illustrate the difference with some simple examples.

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Benchmark Example

I start with an example where life expectancy is constant. The individual \ldots

- ... begins to work at age 20,
- ... works without interruption until 65,
- ... receives a pension until age 80 and then dies.



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Early Retirement in the APG

• Assumptions: Constant Life Expectancy: 80, Retirement age: $65 \rightarrow 60$. • Example

Remarks:

- The NDC reacts automatically to the earlier retirement. The "pension capital" is divided by the higher remaining life expectancy (20 years): 33, 121 = 662, 412/20.
- The new pension level is only 50% (at retirement age 65 it has been 75%).
- The NDC system is balanced.
- Without further deductions the APG would promise a higher pension (44,161), i.e. a replacment rate of 67% instead of 50%.
- The reason: The larger number of pension periods are not taken into account.
- For the example the necessary total deductions would be 25 NB corresponding to an annual deduction of 5.6%.

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Jump in Life Expectancy in the APG — Case 1 $\,$

- Life expectancy 80 \rightarrow 84, retirement age: 65 \rightarrow 68. Example
- Remarks:
 - Reference retirement age: The retirement age such that the pension target of 75% is unchanged. Here 68, since $\frac{15}{45} = \frac{1}{3} \rightarrow \frac{16}{48} = \frac{1}{3}$.
 - The NDC does not need such a reference age, the adjustment is automatic.
 - For the APG this is more complicated. The reference retirement age is needed in order to calculate the new accrual rate. Here: 1.56 = 75/48. The "key formula" has to be continuously adjusted to the changes in life expectancy. In the current case from 45/65/75 to 48/68/75.
 - If the accrual rate is kept constant at 1.67 the initial pension would be 80% (instead of 75%). In this case the system would run a deficit.

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Jump in Life Expectancy in the APG — Case 2

- Life expectancy 80 → 84, retirement age: constant at 65.
 Example
- Remarks:
 - The NDC again reacts automatically to this situation. The replacement rate decreases from 75% to 58%. The budget remains balanced.
 - The APG requires additional deductions. In this case the "actuarial fair" annual deduction is 5.57% (somewhat *lower* than with a life expectancy of 80).

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Constant Increase in Life Expectancy in the APG

- If there is not only a jump in life expectancy but a continuous increase (like in T(t) = T(0) + γt) it is even more difficult.
- There have to be different accrual rates (in the same year) for different cohorts.

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Conclusions

- A well-designed PAYG system like the NDC system bears the promise to deal successfully with demographic developments.
- In contrast to the conventional wisdom, the most appropiate approach is to use period life expectancy and an adjusted growth rate of the wage bill.
- Given that the retirement behavior cannot be controlled, the system needs a reserve fund to deal with short-run imbalances.
- There are a number of additional factors that might also be potential sources of instability for the system: fluctuations in cohort size, fertility age, in the average age of labor market entry, in the age-earnings profiles or in age-specific mortality.
- It is therefore recommendable that a NDC system includes some additional mechanism that adjusts for unforeseen imbalances like the Swedish "automatic balance mechanism NE

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Life Expectancy in the EU

For the EU-countries, e.g., life expectancy at birth is projected to increase over the next 50 years by about 7.5 years which is the main reason behind the projected increase in the old-age dependency ratio from 25.4% in 2008 to 53.5% in 2050 (EPC 2009).
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(Female) life expectancy from 1840 to present Oeppen, J. and Vaupel, J. W. (2002). "Broken Limits to Life Expectancy", *Science* 296 (5570).

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Fig. 1. Record female life expectancy from 1840 to the present [suppl. table 2 (1)]. The linear-regression trend is depicted by a bold black line (slope = 0.243) and the extrapolated trend by a dashed gray line. The horizontal black lines show asserted ceilings on life expectancy, with a short vertical line indicating the year of publication (suppl. table 1). The dashed red lines denote projections of female life expectancy in Japan published by the United Nations in 1986, 1999, and 2001 (): It is encouraging that the U.N. altered its projections or adically between 1999 and 2001.

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- Life expectancy increases in a linear fashion:
 - "Because best-practice life expectancy has increased by 2.5 years per decade for a century and a half, one reasonable scenario would be that this trend will continue in coming decades. If so, record life expectancy will reach 100 in about six decades" (Oeppen and Vaupel, 2002).
- Alternative: Life expectancy reaches a maximum age T^{max}

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- Life expectancy increases in a linear fashion:
 - "Because best-practice life expectancy has increased by 2.5 years per decade for a century and a half, one reasonable scenario would be that this trend will continue in coming decades. If so, record life expectancy will reach 100 in about six decades" (Oeppen and Vaupel, 2002).
- Alternative: Life expectancy reaches a maximum age T^{max}
 - James Fries (1980): Maximum potential life expectancy is normally distributed around 85 with a SD of 7 years.
 - Olshansky and Carnes (2003): "Organisms operate under warranty periods that limit the duration of life of individuals and the life expectancy of populations".

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The orange envelope











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A numerical example to calibrate a pension system

- In a "demographic steady-state": $N(t) = N, T(t) = T, R(t) = R, \forall t.$
- The parameters of the system have to be chosen in a way such that:
 \$\tilde{\tau} = \tilde{q}\tilde{z}\$.
- Based on the case of an "average Austrian pensioner":
 - Retirement age (males): 59.1
 - Life expectancy at the age of 60 (males): 20.7
 - Insured months for new pensioners: 457 (about 38 years)
 - As an approximation this means: the average Austrian pensioner starts working at the age of 20, retires at 60 and dies at 80. Or: R = 40, T = 60 and thus $\hat{z} = \frac{60-40}{40} = \frac{1}{2}$.
 - Furthermore: $\hat{\tau} = 0.3$. Why? The contribution rate is (mostly): 22.8%. The "Bundesmittel" in 2010 have been 8.175 Mio. EUR. This is about a third of the income from contributions ($\frac{8.175}{23.496} = 0.35$) and so 22.8 * 1.35 = 30.7%.

• Therefore a "balanced budget" implies: $\hat{q} = \frac{\hat{\tau}}{\hat{\tau}} = 0.6$.

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Is there a maximum potential life expectancy?

- "Although it is likely that anticipated advances in biomedical technology and lifestyle modification will permit life expectancy to continue its slow rise over the short-term, a repetition of the large and rapid gains in life expectancy observed during the 20th century is extremely unlikely" (Carnes and Olshansky, 2003).
- "First, experts have repeatedly asserted that life expectancy is approaching a ceiling: these experts have repeatedly been proven wrong. Second, the apparent leveling off of life expectancy in various countries is an artifact of laggards catching up and leaders falling behind. Third, if life expectancy were close to a maximum, then the increase in the record expectation of life should be slowing. It is not. For 160 years, best-performance life expectancy has steadily increased by a quarter of a year per year, an extraordinary constancy of human achievement" (Oeppen and Vaupel, 2002).

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Results when life expectancy has an upper limit

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Results when life expectancy has an upper limit

Is there a maximum potential life expectancy? Is it constant over time? And how far away from this limit are we right now? • More quotes

$$T(t) = T(0) + \gamma \cdot t, \text{ for } t < \hat{t}$$

$$T(t) = T(\hat{t}) = T^{max}, \text{ for } t \ge \hat{t}$$



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d(t) when cohort life expectancy reaches a limit



ntroduction	Notation and Budget	NDC systems	APG	Conclusions	Appendix
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Related literature

- Main result in contrast to Torben Andersen (JPubE, 2008): "An indexation of pension ages to longevity may seem a simple and fair solution. This would imply that the relative amount of time spent as contributor to and beneficiary of a social security scheme would be the same across generations with different longevity. [...] However, as is shown in this paper, this solution is not in the feasibility set"
- Reason: TA works with a "two life phases" model where the first life phase has length 1, the second phase has length $\beta \leq 1$ and individuals retire at age $\alpha \leq \beta$.
- The relative retirement age is defined as $\frac{\alpha_t}{\beta_t}$ which is somewhat unusual. Using $\frac{1+\alpha_t}{1+\beta_t}$ leads to the same result as in my framework.

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The German sustainability factor

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The German sustainability factor

Use changes in τ_t and q_t if the dependency ratio z_t changes.

$$\begin{aligned} \tau(t) &= \widehat{\tau} \left[1 + (1 - \alpha) \left(\frac{z_t}{\widehat{z}} - 1 \right) \right] \\ q(t) &= \widehat{q} \left[1 + \alpha \left(\frac{\widehat{z}}{z_t} - 1 \right) \right] \end{aligned}$$

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The German sustainability factor

Use changes in τ_t and q_t if the dependency ratio z_t changes.

$$\tau(t) = \hat{\tau} \left[1 + (1 - \alpha) \left(\frac{z_t}{\hat{z}} - 1 \right) \right]$$
$$q(t) = \hat{q} \left[1 + \alpha \left(\frac{\hat{z}}{z_t} - 1 \right) \right]$$

- Balanced budget in every period
- Works best if life expectancy is constant (or cushioned by a different factor)
- Comprises a continuum of factors: DC ($\alpha = 1$), DB ($\alpha = 0$) and in-between. Each factor implies a different sharing of the demographic burden of boom-and-bust-cycles.
- Not just hypothetical! In Germany a similar factor was introduced in 2003 (with $\alpha = 0.25$)

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"Um die Finanzierung langfristig zu sichern, wird ein Nachhaltigkeitsfaktor eingefuehrt. Dieser basiert bis zum Jahr 2050 auf einem Sollpfad des Anstiegs der periodenbezogenen Lebenserwartung zum Alter 65 des mittleren Szenarios der Statistik Austria. Abweichungen von der mittleren Prognose wirken sich automatisch zur Sicherung der Finanzierbarkeit mit gleicher finanzieller Auswirkung auf Beitragssatz, Steigerungsbeitrag, Antrittsalter, Pensionsanpassung und Bundesbeitrag aus." (ASVG, § 108e, Abs. 9)

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Differences of the Austrian sustainability factor (ASF) to the German case:

- The ASF responds to deviations from forecasts and refers primarily to life expectancy developments. It is a *process* rather than a factor.
- The ASF does not include a mechanism for *automatic* adjustments
- The ASF provides only broad guidelines on how adjustments are to be made. Note that $\alpha = 0.5$ would captures the mixture of two elements (Beitragssatz and Steigerungsbeitrag). In principle it would be possible to include all five elements.
- It is doubtful whether an evenly distributed adjustment is meaningful and intergenerationally fair. E.g.
 "Steigerungsbeitrag" and "Pensionsanpassung" both target the size of pension benefits (the q(t) in the model which stands for the average pension level). Furthermore, who should hear the burden of fluctuations in cohort size?

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Benchmark Case — Constant Life Expectancy

				Def Ac Cont	ined Contrib count (Swed ribution rate	ution len) : 25%	A Target (a Ac	Defined Benefi ccount (Austri it 65 after 45 (crual rate: 1.6	t a) CP): 75% 7%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Year	Age	Inc.	Growth ⊘ wage	Annual Contr.	Total Capital	Pension	Teilgut- schrift	Gesamt- gutschrift	Pension
2010 2011 2012 2013 2053 2054 2055 2056 2057 2068 2059	20 21 22 23	30,000 30,600 31,212 31,836 70,296 71,702	2 % 2 % 2 % 2 % 2 % 2 % 2 % 2 % 2 % 2 %	7,500 7,650 7,803 7,959	7,500 15,300 23,409 31,836	54,852 55,949 57,068 70,957 72,376	500 510 531 1,172 1,195	500 1,020 1,561 2,122 51,550 53,776 54,852	54,852 55,949 57,068 70,957 72,376

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Early Retirement

				Defined Contribution Account (Sweden) Contribution rate: 25% Accrual rate: 1.		Defined Contribution Account (Sweden) Contribution rate: 25% Defined Benefit Account (Austria) Target (at 65 after 45 CP): 7 Accrual rate: 1.67%		Defined Contribution Account (Sweden) Contribution rate: 25		t a) CP): 75% 7%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
Year	Age	Inc.	Growth	Annual	Total	Pension	Teilgut-	Gesamt-	Pension	
			⊘ wage	Contr.	Capital		schrift	gutschrift	(no	
									deduc.)	
2010	20	30,000	2 %	7,500	7,500		500	500		
2011	21	30,600	2 %	7,650	15,300		510	1,020		
.										
:	:	1 :	:	:	:		:	:		
2049	59	64,942	2 %	16,236	649,423		1,082	43,295		
2050	60		2 %		662,412	33,121		44,161	44,161	
2051	61		2 %			33,783			45,044	
.	.					.			.	
:	:		:			:			:	
2068	78		2 %			47,304			63,073	
2069	79		2 %			48,250			64,334	



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Increase in Life Expectancy - Case 1

				Defined Contribution Account (Sweden) Contribution rate: 25%			Defined Benefit Account (Austria) Target (at 68 after 48 CP): 75% Accrual rate: 1.56%		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Year	Age	Inc.	Growth ⊘ wage	Annual Contr.	Total Capital	Pension	Teilgut- schrift	Gesamt- gutschrift	Pension
2010 2011	20 21	30,000 30,600	2 % 2 %	7,500 7,650	7,500 15,300		469 478	469 956	
: 2054	: 64	: 71,702	: 2 %	: 17,925	806,643		: 1,120	50,415	
2055 2056 2057	65 66 67	73,136 74,598 76,090	2 % 2 % 2 %	18,284 18,650 19,023	841,060 876,530 913,084		1,143 1,166 1,189	52,566 54,783 57,068	
2058 2059	68 69		2 % 2 %		931,345	58,209 59,373		58,209	58,209 59,373
2072	: 82		2 %			76.806			: 76.806
2073	83		2 %			78,342			78,342



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Increase in Life Expectancy - Case 2

				Defined Contribution Account (Sweden) Contribution rate: 25%		Defined Benefit Account (Austria) Target (at 68 after 48 CP): 75% Accrual rate: 1.56%			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Year	Age	Inc.	Growth ⊘ wage	Annual Contr.	Total Capital	Pension	Teilgut- schrift	Gesamt- gutschrift	Pension (no deduc.)
2010 2011	20 21	30,000 30,600	2 % 2 %	7,500 7,650	7,500 15,300		469 478	469 956	
2054	64	71.702	2%	17.925	806.643		1.120	50.415	
2055	65		2 %		822,776	43,304		51,423	51,423
2056 2057	66 67		2 % 2 %			44,170 45,053			52,452 53,501
	:		÷			:			:
2072	82		2 %			60,636			72,005
2073	83		2 %			61,849			73,445

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