Increasing Life Expectancy and Pay-As-You-Go Pension Systems

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Abstract

In this paper I study how PAYG pension systems of the notional defined contribution type can be designed such that they remain financially stable in the presence of increasing life expectancy. For this to happen two crucial parameters must be set in an appropriate way. First, the remaining life expectancy has to be based on a cross-section measure and, second, the notional interest rate has to include a correction for labor force increases that are only due to rises in the retirement age which are necessary to "neutralize" the increase in life expectancy. It is shown that the self-stabilization is effective for various patterns of retirement behavior including a linearly rising, a constant, an optimally chosen and a stochastic retirement age.

Keywords: Pension System; Demographic Change; Financial Stability;
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1 Introduction

Pension systems around the world have come under severe pressure from the two-sided demographic development: declining fertility rates and increasing life expectancy. The latter aspect is of particular interest, since it represents an ongoing process with considerable and far-reaching budgetary consequences. For the EU-countries, e.g., life expectancy at birth is projected to increase over the next 50 years by about 7.5 years. This increase is one of the main factors behind the projected rise in the old-age dependency ratio from 25.4% in 2008 to 53.5% in 2050 (EPC, 2009). This development is a particular challenge for pay-as-you-go (PAYG) pension systems. In their traditional organization PAYG systems are based on the implicit assumption of a stationary age structure while ideally they should be designed in such a way as to automatically react to the steady increases in longevity. Recent issues of the OECD's *Pensions at a Glance* deal in detail with the links between life expectancy and retirement and it documents that "around half of OECD countries have elements in their mandatory retirement-income provision that provide an automatic link between pensions and a change in life expectancy" (OECD 2011, p. 81). In fact, the report continues to state that "the rapid spread of such life-expectancy adjustments has a strong claim to be the most important innovation of pension policy in recent years" (OECD 2011, p. 82). In a related paper Edward Whitehouse calls this development a "quiet revolution in pension policy" (Whitehouse 2007, p. 5).

Despite this claim by the OECD that automatic life-expectancy-adjustments are a "quiet revolution" and the "most important innovation" in pension policy there does not exist much empirical and—even less though—theoretical work on the basic functioning, the appropriate design and the main properties of these automatic mechanisms. In this paper I try to fill this gap and focus on the effect of increasing life expectancy in notional defined contribution (NDC) systems (cf. Holzmann & Palmer 2012). NDC systems are of particular interest for a number of reasons. First, they are an increasingly popular variant of the PAYG pension system and—starting with the pioneering work of Sweden—currently around 10 countries have implemented a NDC framework. Second, international institutions like the World Bank, the OECD and the European Commission use the NDC structure as a useful reference point (if not benchmark) to discuss reform proposals and to enhance the transnational portability of pension rights. Third, NDC systems are a natural starting point to analyze the linkage between life expectancy and retirement age since they are explicitly designed in a way such as to react to demographic changes. As will be described in a later section, NDC systems take increases in life expectancy into consideration when the notional capital (i.e. the accumulated contributions) is annuitized. Longer life expectancy will, ceteris paribus, decrease pension benefits, while later retirement will increase them.

While this basic mechanism has been one of the main rationales behind the introduction and propagation of NDC systems, much less is known about the details of its functioning and its optimal design. This, however, is important since self-stabilization will only be achieved if the NDC system is built on accurate construction principles, as will be shown in this paper. For this purpose, I will assume throughout the paper that life expectancy increases in a linear fashion. This is in line with the demographic literature. Furthermore, it has been argued that this linear development is also the best prediction for the behavior of life expectancy for the next 100 years (cf. Oeppen & Vaupel 2002). I show that given this set-up an appropriate design of a NDC system involves the determination of two crucial parameters. First, the "notional interest rate" (or the "rate of return") that specifies how past contributions to the pension system are revalued over time.[1] Second, the measure of "remaining life expectancy" that is used to calculate the first pension benefit at the time of retirement.

The prevailing opinion on this topic is that one should use the growth rate of the wage bill (or, to be precise, the sum of total contributions) as the notional interest rate[2] and the cohort (i.e. forecasted) life expectancy in order to determine the size of the pension annuity.[3] I will show that this conventional wisdom has to be corrected along both dimensions. First, in as far as the measure of life expectancy is concerned I demonstrate that it is sufficient to use periodic life expectancy to calculate the annuity. This is an attractive feature since the determination of remaining life expectancy is then only based on known, cross-section data and does not involve a process of (potentially controversial and politicized) forecasting. Second, I also show that the use of the growth rate of the wage bill is not appropriate in the case of increasing life expectancy. The reason for this is straightforward. When average longevity increases then the cohort-specific retirement age has to rise as well just in order to keep the proportion of retired years to working years

[1] In the generic version of the NDC system the rate at which ongoing pension are adjusted over time is also set equal to the notional interest rate.

[2] "To maintain financial equilibrium, the notional interest rate [...] should be equal to the growth of the covered wage bill, which reflects average wage growth and changes in the labor force" (Chłoń-Domińczak et al. 2012, p. 52). Similar quotes can be found in Börsch-Supan (2003, p. 38) or Palmer (2012, p. 315).

[3] "The second main mechanism, after the correct choice of the notional interest rate, for ensuring the solvency of an NDC scheme involves the application of the correct (future) remaining cohort life expectancy" (Holzmann & Palmer 2012, p. 24). Similar quotes can be found in Chłoń-Domińczak et al. (2012, p. 43) or Palmer (2012, p. 310).

constant. This “neutralizing” postponement of retirement, however, increases by itself the total size of the labor force even if the size of cohorts is constant. Using the growth rate of the wage bill would thus grant an “excessive” rate of return thereby causing a structural deficit of the pension system. The appropriate notional interest rate has to be corrected for this effect. I show that a combination of this adjusted notional interest rate and period life expectancy as the concept to calculate the annuity establishes a self-stabilizing social security system. Whether the budget is balanced in every period or only over time depends on the pattern of retirement behavior as is shown for various assumptions. If the retirement age is always proportional to life expectancy (such that the dependency ratio stays constant over time) then the budget of the NDC system is balanced in every period. The same is true (although only up to a first-order approximation) for the case where the retirement age is assumed to stay constant over time. I also look at the case where the retirement age is chosen in an optimal fashion. For a benchmark case that corresponds to the set-up that is frequently studied in the related literature, the outcome is also a retirement behavior that is associated with a perfectly or approximately balanced budget. Finally, I analyze a situation where the retirement age is a random variable that fluctuates over time. I show that in this case the deficit is on average zero but since the annual deficits will fluctuate around this level the balance will only be achieved in the long run.

The related literature includes empirical analyses, simulation studies and also a small number of theoretical papers. Whitehouse (2007), OECD (2011) and OECD (2012) contain information about the links between life expectancy and various parameters in existing pension systems of OECD countries. Alho et al. (2005) and Auerbach & Lee (2009) use stochastic simulation models in order to evaluate and compare the risk-sharing characteristics of alternative public pension schemes. Since these models allow for a stochastic development of mortality rates they also—implicitly—show how different systems react to changing life expectancy. On the other hand, these papers do not include a systematic discussion on the working and the different design features of automatic life expectancy adjustments. Shoven & Goda (2008) and Heeringa & Bovenberg (2009) are related papers that study how “life expectancy indexation” could be used to stabilize the budget of the public pension systems in specific countries (the US and the Netherlands, respectively). The latter work also contains a stylized model of the use of changes in the retirement age in order to balance increases in longevity. The paper, however, does not compare different formulations of such an indexation. Andersen (2008), on the other hand, uses a two-period model to derive that a “social security system cannot be maintained by simply indexing pension ages to longevity". This is in contrast to the results of the present paper and I will discuss later how to explain this discrepancy. Ludwig & Reiter (2010) study the optimal policy response of a social planner in the presence of demographic shocks. Valdés-Prieto (2000) discusses the ability of NDC system to run on "auto-pilot" but he mainly focuses on the role of changing fertility patterns. Settergren & Mikula (2006) and Palmer (2012) present results that are related to the ones of the present paper although they derive them in a different framework (involving the "turnover duration") and they also lack an explicit treatment of the case with constantly increasing life expectancy.

The paper is structured as follows. In the next section I discuss the assumption about the development of life expectancy and I present the structure of a general PAYG system. In section 3 I then describe and formalize the main features of a NDC system and I discuss the two main parameters: the notional interest rate and the measure of life expectancy. In section 4 I show how one can determine these two parameters in order to design a NDC system that is self-stabilizing for various assumptions concerning retirement behavior. Section 5 concludes.

2 The Model

2.1 Basic set-up

I work with a model in continuous time (cf. Bommier & Lee 2000). In every instant of time t a generation is born that has size $N(t)$ and lives for $T(t)$ years. All members of generation t start to work as soon as they are born and they remain in the labor market for $R(t)$ periods, earning a wage $W(t+a)$ during each of these working periods $a \in [0, R(t)]$.[4] Thereafter, they receive a pension benefit $P(t,a)$ in each period of retirement $a \in [R(t), T(t)]$. While working, individuals pay contributions to the PAYG pension system at rate $\tau(t+a)$. The (relative) pension level is defined as $q(t,a) = \frac{P(t,a)}{W(t+a)}$ and the growth rate of wages is denoted by $g(t)$, i.e. $W(t) = W(0)e^{\int_0^t g(s)\,\mathrm{d}s}$.

As far as the development of life expectancy is concerned I make a number of assumptions that allow for simple and intuitive expressions. First, I focus on a representative member of generation t and I abstract from all *intra*generational differences. In particular, I assume that all members of a generation reach their cohort life expectancy $T(t)$. Second, I assume that the retirement age is non-decreasing over time, i.e. $R(t+dt) \geq R(t)$. This

[4] I abstract here from the existence of an age-earnings profile. At each point in time all workers are assumed to earn the same wage.

makes it possible to express all aggregate values in a compact form without the use of "indicator variables". Third, I assume that the development of life expectancy follows a deterministic process that is perfectly known by all agents. A number of important issues that arise in the case of aggregate and idiosyncratic longevity risk are discussed in Alho et al. (2012). Fourth, life expectancy is assumed to increase linearly over time:

$$T(t) = T(0) + \gamma \cdot t, \tag{1}$$

where $0 \leq \gamma < 1$. This assumption is in line with the empirical literature. Oeppen & Vaupel (2002), e.g., analyze "record female life expectancy" (i.e. the highest value for female life expectancy reported in any country for which data are available) from 1840 to 2000 and they show that it follows an almost perfect linear development with a slope parameter of 1/4.[5] This is confirmed by Lee (2003) who refers to a number of studies that have found a linear trend in life expectancy for a large sample of industrial countries with slope parameters between 0.15 and 0.25. I deal with the case where life expectancy is assumed to reach a biologically determined maximum age in a companion paper.

In order to be able to distinguish clearly between the viewpoint of generation t (i.e. the one born in t) and the outlook of the pension system in period t I introduce two further variables. $\widetilde{T}(t)$ stands for period life expectancy, i.e. the highest age observed in period t. $\widetilde{R}(t)$, on the other hand, denotes the number of working years of the generation that retires in period t. In general it will be the case that $\widetilde{T}(t) \neq T(t)$ and $\widetilde{R}(t) \neq R(t)$.

Period life expectancy in period t can be calculated from cohort life expectancy by the following relation: $T(t - \widetilde{T}(t)) = \widetilde{T}(t)$. It comes out as:

$$\widetilde{T}(t) = \frac{1}{1+\gamma} T(t). \tag{2}$$

The increase in period life expectancy is given by $\frac{d\widetilde{T}(t)}{dt} = \frac{\gamma}{1+\gamma}$. A value of $\frac{\gamma}{1+\gamma} = 1/4$ thus implies $\gamma = 1/3$, while $\frac{\gamma}{1+\gamma} = 1/5$ corresponds to $\gamma = 1/4$. For the following numerical examples I will use the latter value which is about the mid-point of the estimates reported in Lee (2003).[6]

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[5] Their data refer to the "mean age at death under current mortality conditions", i.e. to *period* life expectancy. Equation (1), however, refers to the development of *cohort* life expectancy. I will show later that the estimated slope in Oeppen & Vaupel (2002) implies a value of $\gamma = 1/3$.

[6] In the demographic literature there exist a number of papers that have studied the relation between different life expectancy concepts in standard mortality models. Goldstein (2006) and Missov & Lenart (2011) show, e.g., that under special assumptions (like a "linear shift model" or a Gompertz mortality model with constant yearly improvements at all ages) period and cohort life expectancy increase in a

2.2 Budget of the pension system

The total size of the active population $L(t)$, of the retired population $B(t)$ and the resulting dependency ratio $z(t)$ are given by:

$$L(t) = \int_0^{\widetilde{R}(t)} N(t-a)\,\mathrm{d}a, \tag{3}$$

$$B(t) = \int_{\widetilde{R}(t)}^{\widetilde{T}(t)} N(t-a)\,\mathrm{d}a, \tag{4}$$

$$z(t) = \frac{B(t)}{L(t)}. \tag{5}$$

The total revenues $I(t)$ and the total expenditures $O(t)$ of the pension system in a certain period t are:

$$I(t) = \int_0^{\widetilde{R}(t)} \tau(t)W(t)N(t-a)\,\mathrm{d}a, \tag{6}$$

$$O(t) = \int_{\widetilde{R}(t)}^{\widetilde{T}(t)} P(t-a,a)N(t-a)\,\mathrm{d}a. \tag{7}$$

The total deficit (or surplus) is denoted by:

$$D(t) = O(t) - I(t), \tag{8}$$

while the deficit ratio d_t is written as:[7]

$$d(t) = \frac{O(t)}{I(t)}. \tag{9}$$

A balanced budget thus requires that $D(t) = 0$ or $d(t) = 1$, $\forall t$.

2.3 Demographic Steady State

For the following analysis it is helpful to use a steady state (or rather a "balanced growth path") as a reference point. It is a "triple stationary state" that involves the demography,

linear fashion. The formulation in (1) can thus also be understood as a short-cut for a fully-fledged model with mortality rates.

[7]Alternatively, one could also use the deficit relative to "national income" (i.e. $\frac{D(t)}{W(t)L(t)}$) or simply the deficit-revenues ratio ($\frac{D(t)}{I(t)} = d(t) - 1$). In general, it does not matter which concept is used since they all lead to qualitatively similar patterns.

the economy and the pension system. First, the demographic situation is assumed to be constant over time, i.e. $N(t) = \overline{N}$ and $T(t) = \overline{T}$. Second, the economy grows at a constant rate ($g(t) = \overline{g}$). Third, also the main parameters of the pension system do not change and the retirement age, the contribution rate and the average pension level are constant, i.e.: $R(t) = \overline{R}$, $\tau(t) = \overline{\tau}$ and $\overline{q}(t) = \frac{\overline{P}(t)}{W(t)} = \frac{\int_{\overline{R}}^{\overline{T}} P(t-a,a)\,\mathrm{d}a.}{W(t)} = \overline{q}$. From (3) and (4) it then follows that also $z(t) = \frac{B(t)}{L(t)} = \frac{\overline{T}-\overline{R}}{\overline{R}} \equiv \overline{z}$ is constant over time, where $\overline{z}$ is the steady state dependency ratio and also the ratio of the average years a person stays in pension to the average years he or she is in work. A permanently balanced budget then requires that in the steady state the following relation must hold:

$$\bar{\tau} = \bar{q}\bar{z}. \tag{10}$$

Each society may choose its preferred values for the steady state contribution rate, pension level and retirement age, as long as condition (10) is fulfilled and as long as $\overline{\tau} < 1$.

3 A Notional Defined Contribution Pension System

3.1 Formal expression of a NDC system

Thus far I have left open how the pension levels $P(t-a,a)$ of the various retired cohorts at a certain period t are determined. It is in this area that one observes the biggest cross-national differences and also the main rift between defined benefit and defined contribution systems. In this paper I focus on the NDC system that has been implemented first in Sweden and has later been also adapted by a number of additional countries like Italy, Poland, Latvia, Mongolia, Turkmenistan etc. It is now also often used as a benchmark PAYG model by international institutions like the World Bank (Holzmann & Hinz 2005), the OECD (2011) or the European commission (EPC, 2009).[8]

The first crucial feature of every NDC system is that the contribution rate is fixed at $\tau(t) = \overline{\tau}$ for all periods. In Sweden, e.g., each insured person pays 16% of its earnings (up to a ceiling) as contributions to the notional account. The value of the "deposits" on this account grow with the "notional interest rate" $\rho(t)$ which in Sweden is defined as the average growth rate of wages. The total value in this account is called the notional capital $K(t,a)$ that generation t accumulates over the working periods $a \in [0, R(t)]$. The

[8]Detailed descriptions can be found in Palmer (2012) or Chłoń-Domińczak et al. (2012).

final amount of this notional capital (before it is turned into an annuity) is given by:

$$K(t,R(t)) = \int_0^{R(t)} \overline{\tau} W(t+a) e^{\int_{t+a}^{t+R(t)} \rho(s)\,\mathrm{d}s}\,\mathrm{d}a, \tag{11}$$

where the cumulative factor $e^{\int_{t+a}^{t+R(t)} \rho(s)\,\mathrm{d}s}$ indicates how the contribution $\overline{\tau}W(t+a)$ that is paid into the pension system in period $t+a$ is revalued when calculating the final amount of the notional capital in period $t+R(t)$ (the period of retirement). The specification of the notional interest rate is one of the crucial parameters in a NDC system and I will later discuss various possibilities how it can (and should) be defined.

The first pension that is received by generation t in period $t+R(t)$ is given by:

$$P(t,R(t)) = \frac{K(t,R(t))}{\Gamma(t,R(t))}, \tag{12}$$

where the notional capital for generation t is transformed into an annuity by using the remaining life expectancy $\Gamma(t,R(t))$ of generation t at the retirement age $R(t)$.[9] The conceptual measure that is used to calculate the remaining life expectancy (period vs. cohort life expectancy) is another crucial factor for the specification of a NDC system. I will come back to this issue below.

Existing pensions are adjusted according to:

$$P(t,a) = P(t,R(t)) e^{\int_{t+R(t)}^{t+a} \vartheta(s)\,\mathrm{d}s}, \tag{13}$$

for $a \in [R(t), T(t)]$ and where $\vartheta(s)$ stands for the adjustment rate in period s and the cumulative adjustment factor $e^{\int_{t+R(t)}^{t+a} \vartheta(s)\,\mathrm{d}s}$ indicates how the first pension $P(t,R(t))$ received by generation t at age $R(t)$ is adjusted to give the pension payment in period $t+a$.

Inserting equations (11), (12) and (13) into (7) and assuming $\vartheta(t) = \rho(t)$ leads to the following expression of expenditures (see appendix A.1):

$$O(t) = \overline{\tau} W(t) \int_{\widetilde{R}(t)}^{\widetilde{T}(t)} \frac{\int_0^{R(t-a)} \left[e^{\int_{t-a+b}^{t} (\rho(s)-g(s))\,\mathrm{d}s} \right] \mathrm{d}b}{\Gamma(t-a, R(t-a))} N(t-a)\,\mathrm{d}a. \tag{14}$$

The question is, whether one can find definitions for $\rho(t)$ and $\Gamma(t,R(t))$ such that (14) de-

[9] In order to keep the analysis simple I abstract here from the issue of front-loading as is currently used in Sweden. Under this regime a real growth rate of 1.6% is used to "frontload" part of the expected pension adjustments thereby increasing the initial pension. Existing pensions are then only adjusted with the difference between the actual growth rate and this stipulated growth rate of 1.6%.

velops in line with the revenues of the pension system given by (6), i.e. $I(t) = \overline{\tau}W(t)L(t)$.

In order to focus clearly on the issue of increasing life expectancy, I will assume in the following that the size of birth cohorts is constant:

$$N(t) = \overline{N}. \tag{15}$$

3.2 Crucial choices in NDC systems

The crucial factors that have to be defined at the outset for a functioning NDC system are thus the notional interest rate $\rho(t)$, the adjustment factor $\vartheta(t)$ and the measure of life expectancy that is used to specify the remaining life expectancy $\Gamma(t, R(t))$.

To start with the latter there exist two variants. One can either use period (or cross-section) life expectancy $\widetilde{T}(t + R(t))$ or cohort (or forecasted) life expectancy $T(t)$ to calculate the annuity payment for generation t. This means either:

$$\Gamma(t, R(t)) = \widetilde{T}(t + R(t)) - R(t) \tag{16a}$$

or

$$\Gamma(t, R(t)) = T(t) - R(t). \tag{16b}$$

In as far as the notional interest rate is concerned there are two possible methods of indexation that are often discussed in the literature and that are used in real-world pension systems: an indexation with the growth rate of average wages and one with the growth rate of the wage bill (or rather the growth rate of the sum of contributions). I will only discuss methods where the notional interest rate and the adjustment rate are identical, i.e. where $\vartheta(t) = \rho(t)$. The formulas are given by:

$$\rho(t) = g^W(t) \tag{17a}$$

or

$$\rho(t) = g^W(t) + g^L(t), \tag{17b}$$

where $g^W(t) \equiv \frac{\dot{W}(t)}{W(t)}$, $g^L(t) \equiv \frac{\dot{L}(t)}{L(t)}$, $\dot{W}(t) \equiv \frac{dW(t)}{dt}$ and $\dot{L}(t) \equiv \frac{dL(t)}{dt}$. Using the definitions for $W(t) = W(0)e^{\int_0^t g(s)\,\mathrm{d}s}$ and $L(t) = \int_0^{\widetilde{R}(t)} N(t-a)\,\mathrm{d}a = \widetilde{R}(t)\overline{N}$ (due to assumption (15)) one can calculate that:

$$g^W(t) = g(t), \tag{18}$$

$$g^L(t) = \frac{d\widetilde{R}(t)}{dt} \frac{1}{\widetilde{R}(t)}. \tag{19}$$

Given that life expectancy is assumed to increase linearly according to (1) it is a natural benchmark to assume that the retirement age is a fixed proportion of life expectancy and thus also increases in a linear fashion. In particular, assume that:

$$R(t) = \mu T(t), \tag{20}$$

where $0 < \mu < 1$. One can use the relation that $R(t - \widetilde{R}(t)) = \widetilde{R}(t)$ to derive that $\widetilde{R}(t) = \frac{\mu}{1+\gamma\mu} T(t)$. Setting the retirement age according to (20) thus leads to a situation where the dependency ratio $z(t)$ is constant and given by $z(t) = z = \frac{\widetilde{T}(t) - \widetilde{R}(t)}{\widetilde{R}(t)} = \frac{1-\mu}{\mu(1+\gamma)}$.[10] The period-specific retirement age $\widetilde{R}(t)$ increases linearly with $\frac{d\widetilde{R}(t)}{dt} = \frac{\gamma\mu}{1+\gamma\mu}$ and finally $g^L(t) = \frac{d\widetilde{R}(t)/dt}{\widetilde{R}(t)} = \frac{\gamma}{T(t)} = \frac{\gamma}{(1+\gamma)\widetilde{T}(t)}$. This result delivers an important insight. If life expectancy increases and if every cohort prolongs its working life in such a manner as to counter this increase and to keep the dependency ratio constant then there will be a continuous increase in the size of the labor force even if the size of cohorts remains constant. This is simply a consequence of the fact that each cohort postpones its retirement by a little bit, as specified in (20). In fact, this behavior seems like the "natural" and most appropriate reaction to the increase in life expectancy. Taking this into consideration it also appears unjustified that this "necessary" and "appropriate" increase in the labor force should lead to a higher notional interest rate as would be the case if one uses the growth rate of the wage bill (cf. (17b)) as the relevant concept. It seems more reasonable to propose a new concept that defines the notional interest rate as the growth rate of the wage bill *adjusted* for the necessary increase in retirement age due to increasing life expectancy. This leads to the third concept for the determination of the notional interest rate and the adjustment factor that I will study in the following:

$$\rho(t) = g^W(t) + g^L(t) - \frac{\gamma}{(1+\gamma)\widetilde{T}(t)}. \tag{17c}$$

[10] If there exists a specific target $\bar{z}$ for the dependency ratio one has to set $\mu = \frac{1}{1+\bar{z}(1+\gamma)}$ in order to achieve $z(t) = \bar{z}$.

4 A self-stabilizing NDC system

Looking at the expression for total pension expenditures $O(t)$ in (14) it does not seem obvious whether it is at all possible to find rules for the determination of the notional interest rate and remaining life expectancy such as to implement a NDC system that has a budget that is balanced (at least over the long run). The main challenge is to guarantee financial sustainability for a large (or even an *arbitrary*) pattern of retirement ages. In the following I will therefore look at various assumptions concerning the retirement choices and their effects on the long-run budget of the NDC system.

4.1 Retirement age is proportional to life expectancy

As a useful benchmark case I look first at the situation where the retirement age is a fixed proportion of life expectancy as specified in (20) and repeated here:

$$R(t) = \mu T(t). \tag{21a}$$

As said above, this seems to be a "natural" and intergenerational equitable reaction to the continuous increase in life expectancy where for every generation the proportion of retirement years to working years stays constant at $\frac{T(t)-R(t)}{R(t)} = \frac{1-\mu}{\mu}$. As previously mentioned, in this case the labor force grows with $g^L(t) = \frac{\gamma}{(1+\gamma)\widetilde{T}(t)}$ and thus the notional interest rate (17c) is given by $\rho(t) = g(t)$ which is the same as using average wage growth (cf. (17a)).

The following proposition specifies how to design a NDC system in this situation such that it is compatible with long-run sustainability.

Proposition 1 *Assume that life expectancy increases in a linear fashion and that the size of birth cohorts is constant. If the retirement age is a fixed proportion of life expectancy as assumed in* (21a) *then a NDC system leads to a balanced budget if the following two conditions are fulfilled: (i) The notional interest rate and the adjustment factor are equal to the growth rate of average wages* (17a) *or the adjusted growth rate of the wage bill* (17c) *and (ii) the remaining life expectancy is calculated by using period life expectancy as in* (16a)*.*

Proposition 1 is surprising since it contradicts claims about the most reasonable set-up of a NDC system that can be found in the related literature.[11] There it is stated that

[11]Examples for this have been quoted in the introduction. Proposition 1 is in contrast to the finding of

Table 1: The deficit-ratio for a retirement age that is proportional to life expectancy

	(1)	(2)	(3)
	Notional Interest Rate — Growth Rate of:		
	Average Wages	Wage Bill	Adjusted Wage Bill
Period Life Expectancy	1	$\approx 1+\frac{\gamma}{2}$	1
Cohort Life Expectancy	$\frac{1}{1+\gamma}$	$\approx 1-\frac{\gamma}{2}$	$\frac{1}{1+\gamma}$

Note: The table shows the period deficit ratio $d(t)=\frac{O(t)}{I(t)}$ for various assumption about the notional interest rate and the measure of remaining life expectancy. Period [cohort] life expectancy is defined as in (16a) [(16b)]. The notional interest rate is given by one of the expressions in (17a), (17b) and (17c), respectively. Furthermore, it is assumed that retirement age increases in a linear fashion according to $R(t)=\mu T(t)$.

the most appropriate design would involve a combination of cohort life expectancy (cf. (16b)) and the unadjusted growth rate of the wage bill (cf. (17b)).

The proof of the proposition can be found in appendix A.2. In Table 1 I summarize the budgetary outcomes for all combinations of assumptions (17a) to (17c) and assumptions (16a) and (16b). As documented in Table 1 if one uses the notional interest rate $\rho(t)=g(t)$ together with period life expectancy (16a) then the budget of the pension system is always balanced, i.e. $D(t)=0$ and $d(t)=1$ for every period.

For the stability of the system it is crucial that one uses the adjusted growth rate of the wage bill (17c) and not the unadjusted wage bill growth rate (17b) as is often suggested in the literature. For $\gamma>0$ the latter concept would prescribe a higher notional interest rate ($\rho(t)=g(t)+\frac{\gamma}{(1+\gamma)\widetilde{T}(t)}$) which would cause a permanent deficit of the pension system. In particular, in appendix A.2 I show that the deficit ratio in this case is approximately equal to $d(t)=1+\frac{\gamma}{2}$. This is a non-trivial amount. For a realistic value of γ it would amount to a permanent deficit ratio of about 12.5%. The reason for this imbalance is that such a system would grant an extra rate of return due to the permanent increase in the

Andersen (2008) that an "indexation of pension ages to longevity may seem a simple and fair solution" which, however, "is not in the feasibility set" (p. 634f.). The main reason for the difference between the results stems from the use of different notions of "proportionality". Andersen (2008) uses a rather unconventional definition of the relative span of retirement which is not defined as the length of the retirement period to the total length of working life but rather only to the years of work since the "middle age".

labor force which is, however, just a necessary reaction to the increases in life expectancy and should be neglected when determining the rate of return.

On the other hand, if one uses adjusted wage bill growth (17c) but cohort life expectancy (16b) instead of period life expectancy (16a) then the deficit ratio is given by $d(t) = \frac{1}{1+\gamma}$ and the pension system runs a permanent surplus. The use of cohort life expectancy (as frequently recommended for NDC systems) is thus “overambitious” as it will lead to excessively small annuities that cause a permanent surplus in the budget. It is sufficient to use period life expectancy if this is combined with the appropriate notional interest rate (17c).[12]

4.2 Retirement age stays constant

It is certainly an optimistic scenario to assume that retirement age always adjusts in lock-step to the increases in life expectancy. As the opposite (very “pessimistic”) extreme one could also assume that the retirement remains constant despite the advances in longevity:

$$R(t) = \overline{R}. \tag{21b}$$

Note that then the labor force is constant ($g^L(t) = 0$) and thus the notional interest rate (17c) is given by $\rho(t) = g(t) - \frac{\gamma}{(1+\gamma)\widetilde{T}(t)}$. The following proposition contains the main result.

Proposition 2 *Assume that life expectancy increases in a linear fashion and that the size of birth cohorts is constant. If the retirement age is fixed as assumed in* (21b) *then a NDC system leads to an approximately balanced budget if the following two conditions are fulfilled: (i) The notional interest rate and the adjustment factor are equal to the adjusted growth rate of the wage bill* (17c) *and (ii) the remaining life expectancy is calculated by using period life expectancy as in* (16a).

The proof of proposition 2 can be found in appendix A.3 and the deficit ratios corresponding to the various combinatinos of assumptions are collected in Table 2. One can derive that the combination of a notional interest rate set according to (17c) and a remaining life expectancy that is based on period life expectancy as in (16a) leads to a

[12]Since the use of (16b) instead of (16a) leads to a permanent surplus while the use of (17b) instead of (17c) to a permanent deficit one might want to know what happens if one were to combine (16b) and (17b). The answer is that in this case the deficit ratio is approximately $1 - \frac{\gamma}{2}$ (see appendix A.2) and one faces a permanent surplus.

Table 2: The deficit-ratio for a constant retirement age

	(1)	(2)	(3)
	Notional Interest Rate — Growth Rate of:		
	Average Wages	Wage Bill	Adjusted Wage Bill
Period Life Expectancy	$\approx 1+\frac{\gamma}{2}$	$\approx 1+\frac{\gamma}{2}$	≈ 1
Cohort Life Expectancy	$\approx 1-\frac{\gamma}{2}$	$\approx 1-\frac{\gamma}{2}$	$\approx 1-\gamma$

Note: The table shows the period deficit ratio $d(t) = \frac{O(t)}{I(t)}$ for various assumption about the notional interest rate and the measure of remaining life expectancy. Period [cohort] life expectancy is defined as in (16a) [(16b)]. The notional interest rate is given by one of the expressions in (17a), (17b) and (17c), respectively. Furthermore, it is assumed that retirement age is constant at $R(t) = \overline{R}$.

deficit ratio given by:

$$d(t) = \frac{\overline{R}}{\widetilde{T}(t)} \left(\frac{(2+\gamma)\ln(1+\gamma)}{2\gamma} - 1 \right) + 1 \approx 1,$$

where the approximation is around $\gamma = 0$. So in this case the combination of (17c) and (16a) leads to a budget that is approximately balanced in every period.[13]

It is interesting to study here what would happen if one uses a "conventional" notional interest rate with the growth rates of either average wages (17a) or the wage bill (17b). These two cases are now identical (since $g^L(t) = 0$) implying a notional interest rate of $\rho(t) = g(t)$. Using period life expectancy one can calculate that:

$$d(t) = \frac{(1+\gamma)\ln(1+\gamma)}{\gamma} \approx 1 + \frac{\gamma}{2}.$$

This seems to confirm the belief that the use of period life expectancy is not enough to keep a NDC system in balance. The use of cohort life expectancy (16b), however, as is often suggested as the better alternative is also not appropriate as it leads to a permanent

[13] One has to note, however, that the calculations abstract from the existence of a minimum pension. If social security legislation prevents a fall of the relative pension level $q(t)$ below some minimum level q^{min} then a fixed retirement age will lead to an increasing deficit even in a NDC system.

surplus.

$$d(t) = \frac{\ln(1+\gamma)}{\gamma} \approx 1 - \frac{\gamma}{2}.$$

The punchline of this consideration is that in the case of constant retirement ages and a notional interest rate $\rho(t) = g(t)$ both methods of calculating the remaining life expectancy for annuitization at the time of retirement lead to an unbalanced budget. The first method is too "generous" causing persistent deficits while the second method is too "harsh" leading to ongoing surpluses.

While the use of the adjusted growth rate of the wage bill as the notional interest rate steers the system towards financial sustainability, the balance is not exact as stated in proposition 2. The error of approximation, however, is rather small. Using $\gamma = 1/4$, $\overline{R} = 45$ and $T(0) = 60$ (such that $z(0) = 1/3$) one gets an exact value of $d(0) = 1.00389$, i.e. expenditures exceed revenues by 0.4%. This is the amount that has to be channeled to the pension system from the general budget in order to keep it stable. This is a rather modest amount, in particular when compared to the alternative methods. The use of period life expectancy and the (unadjusted) growth rate of the wage bill leads to a permanent deficit ratio of 11% while the use of cohort life expectancy gives rise to a permanent surplus of 10.7%.

Taking propositions 1 and 2 together, one can thus conclude that only the combination of the adjusted growth rate of the wage bill (17c) and period life expectancy (16a) leads to an (approximately) balanced budget if the retirement age is set according to $R(t) = \mu T(t)$ or $R(t) = \overline{R}$. In fact, it can be shown that the same result holds if retirement age is a linear combination of (21a) and (21b), i.e. $R(t) = \varsigma \mu T(t) + (1-\varsigma)\overline{R}$, for $0 \leq \varsigma \leq 1$. In this case (where the retirement age increases linearly with $\frac{dR(t)}{dt} = \varsigma\mu\gamma$) the budget is again almost balanced for all values of ς.

4.3 Retirement age is optimally chosen

So far I have assumed that the retirement age increases linearly as a reaction to a linearly increasing life expectancy. In this section, I want to show that under certain assumptions this can in fact be regarded as the optimal behavior of utility-maximizing individuals. I will also show that even for other optimal rules the budget of the NDC system will stay approximately balanced.

In order to model the optimal choice of retirement I follow the literature that has dealt with this issue (cf. Sheshinski 1978, Crawford & Lilien 1981, Bloom et al. 2003, Kalemli-

Ozcan & Weil 2010). I assume that agents maximize their lifetime utility, choosing how much to consume in each period and how long to work. I ignore the effects of any eventual bequest motives, of the family structure and of all possible sources of uncertainty. The intertemporal utility function for the representative member of generation t is given by:

$$\mathbb{U} = \int_0^{T(t)} e^{-\delta a} U(C(t,a))\,\mathrm{d}a - \int_0^{R(t)} e^{-\delta a} V(T(t),a)\,\mathrm{d}a, \tag{22}$$

where $C(t,a)$ is the level of consumption of cohort t at age a, $V(T(t),a)$ captures the disutility of work of generation t (with life expectancy $T(t)$) at age a and δ is the rate of time preference. The stock of assets $A(t,a)$ evolves according to:

$$\frac{dA(t,a)}{da} = rA(t,a) + \chi(t,a)(1-\tau)W(t+a) + (1-\chi(t,a))P(t,a) - C(t,a), \tag{23}$$

where r is the interest rate and $\chi(t,a)$ is an indicator variable with the value $\chi(t,a) = 1$ if generation t is working and $\chi(t,a) = 0$ if the cohort is retired. Agents choose their consumption paths and their retirement age $R(t)$ subject to the conditions that $C(t,a) > 0$ and $A(t,a) \geq 0$ (no borrowing).

As is done in many papers of the related literature I focus here on the case where wages are constant and where both the interest rate and the discount rate are zero. In this case the first-order condition for consumption implies that each generation has a flat consumption profile (i.e. $C(t,a) = \overline{C}(t),\ \forall a$). Lifetime income $\Omega(t)$ is then given by $\Omega(t) = \overline{C}(t)T(t)$ and the first-order condition concerning the retirement age can be written as:

$$V(T(t),R(t)) = \frac{\partial \Omega(t)}{\partial R(t)} U'(\overline{C}(t)). \tag{24}$$

Each generation will work as long as the costs of the additional period of work are smaller than the benefit of this effort. The optimal retirement age thus depends crucially on the assumption concerning the disutility function $V(T(t),a)$.

A benchmark in the literature is the situation where $V(T(t),a)$ is assumed to be homogeneous of degree 0, i.e. $V(\alpha T(t), \alpha a) = V(T(t),a)$ for $\alpha > 0$. This amounts to the assumption that "health status at each age improves proportionately with life expectancy. [...] The health status and disutility of someone working at age 45 who has a life expectancy of 60 is the same as the health status and disutility of someone working at age 60 who has a life expectancy of 80" (Bloom et al. 2007, p. 96). If one assumes log utility of consumption and the combination of assumptions (17c) and (16a) it can be shown (see appendix A.4) that in this situation the optimal retirement age is proportional to life expectancy, i.e. $R^*(t) = \mu T(t)$, as assumed in section 4.1. In fact, in the appendix I show that the proportional retirement age is also the optimal strategy for all other combinations of assumptions concerning the notional interest rate and the life expectancy concept. However, as already discussed in section 4.1, it is only the combination of (17c) (or (17a)) and (16a) that is also associated with a balanced budget.

A proportional increase in the retirement age as a reaction to an increase in life expectancy is, however, no longer optimal if the disutility of labor does not change pari passu with increasing life expectancy, i.e. if $V(T(t), a)$ is not homogeneous of degree 0. A simple assumption that captures this phenomenon is that the disutility increases linearly in age, i.e. $V(T(t), a) = \upsilon a$. In appendix A.4 I show that in this case:

$$R^*(t) = \sqrt{\frac{T(t)}{\upsilon}}, \tag{21c}$$

i.e. the retirement age increases less than proportionally to the increase in life expectancy. Nevertheless, it can be shown that even in this case the pension design given by (17c) and (16a) leads to an approximately balanced budget while all other combinations of the notional interest rates and the concept of remaining life expectancy are associated with budgetary imbalances.

4.4 Retirement age is random

In the previous section I have used a stylized model to show that a NDC system that is designed in the right way can remain balanced when individuals choose their retirement age in an optimal fashion. In a realistic setting, however, one had to take into account that the factor prices are not constant but rather fluctuating and that also the disutility of labor is likely to change over time. The outcome of such a more realistic model will likley lead to patterns of the retirement age that are less regular than the ones given in (21a) or (21c). Instead of using such a more complex model I want to analyze the reaction of the NDC system to a more erratic pattern of retirement choices in a different way. In particular, I simply assume that — for whatever reason — the retirement age follows a stochastic pattern and I use numerical simulations to study the budgetary consequences of this pattern.[14] I have experimented with various assumptions. Here I present the

[14]Details about the simulations can be found in appendix B. In particular, the simulations are based on a discrete-time version of the model and therefore the variables $R(t)$, $d(t)$ etc. should be understood

results of one example where I assume that the cohort-specific retirement age is a random variable that is uniformly distributed around the value given by (21a).

Figure 1 shows one path for $R(t)$ under this assumption. The pattern is not meant to be realistic but rather represents an extreme scenario that allows to study the self-stabilizing possibilities of NDC systems. Figure 2 reports the pattern of $d(t)$ that emerges in this scenario if one uses the adjusted wage bill growth and period life expectancy as its crucial parameters.

As expected the pension system is unbalanced in almost every year and the deficit ratio fluctuates with a minimum and maximum of 0.92 and 1.1, respectively, and a standard deviation of 0.038. Over time, however, the surpluses and the deficits counteract each other and the average deficit ratio over the 150 year period is 0.999. In order to smooth out these fluctuations the pension system will need to establish a reserve fund. The development of the assets of this fund will then not only depend on the exact pattern of retirement behavior but also on the size of the interest rate and on the exact sequence of surpluses and deficits. In general, however, since the deficit ratio fluctuates around 1, the system will have a tendency to balance over time. This is in contrast to NDC systems that are based on different combinations of the notional interest rate and remaining life expectancy. Using the (unadjusted) growth rate of the wage bill and period (cohort) life expectancy leads to an average deficit ratio of 1.12 (0.89). This outcome is not due to the specific numerical example reported in Figure 1 but it has been shown for a large number of different simulations. For any of the many simulations I have run the average deficit ratio has been very close to 1 if one uses the appropriate design of the NDC system.

5 Conclusion

In this paper I have studied how to design a NDC pension system that is able to stabilize its budget in the presence of increasing life expectancy. I have shown that the financial sustainability depends on the appropriate determination of two parameters: the notional interest rate and the measure that is used to calculate remaining life expectancy. A combination of the growth rate of an adjusted wage bill as notional interest rate and period life expectancy will lead to a balanced budget for a large variety of possible retirement behaviors. These findings are a challenge to the conventional wisdom on the appropriate design of NDC systems and none of the countries that are currently organized in such a

as R_t, d_t etc. I do not make these notational substitutions here in order to keep the formulas and figures in line with the rest of the text.

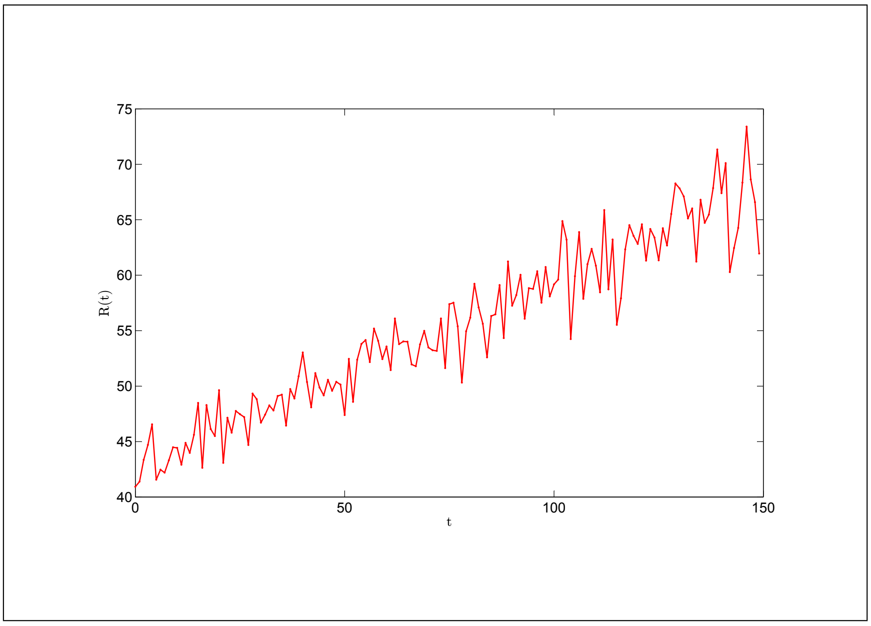

Figure 1: The figure shows one path of the cohort-specific retirement age when $R(t)$ is a random variable that is uniformly distributed around $R(t) = \mu T(t)$. The simulation has been run on a monthly basis while the graph shows the annual average. The parameters were chosen as follows: $\overline{\tau} = 0.25$, $\mu = 0.71$, $\gamma = 0.25$, $T(0) = 60$ and $R(t)$ fluctuates between 0.75 and 1.25 of the reference value.

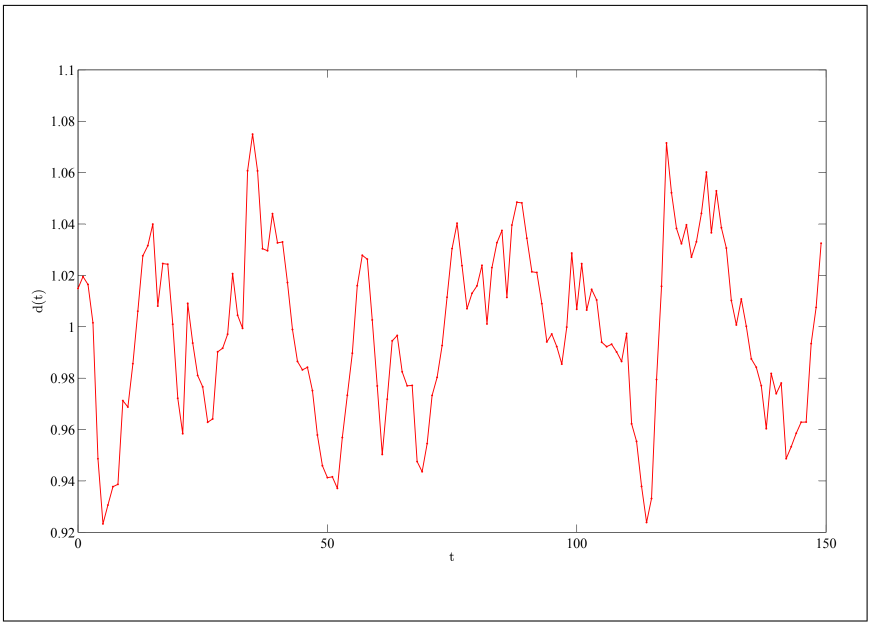

Figure 2: The figure reports the development of $d(t)$ when the cohort-specific retirement age is given by the pattern shown in figure 1. The simulation has been run on a monthly basis while the graph shows annual values. The parameters were chosen as follows: $\overline{\tau} = 0.25$, $\mu = 0.71$, $\gamma = 0.25$ and $T(0) = 60$.

way uses the combination of parameters that suggests itself in the modeling framework of this paper. These findings might thus be useful for the refinement of existing or the construction of future NDC systems.

The focus of this paper has been to analyze the impact of increasing life expectancy on the stability of PAYG pension systems. Therefore, I have abstracted from all other economic and demographic factors that might also be potential sources of instability for the system. First and foremost this concerns changes along the second demographic dimension: the size of the birth or working cohorts $N(t)$. Different fertility scenarios have already been studied in the related literature (cf. in particular Valdés-Prieto 2000). The main finding is that non-monotonic shifts in the development of cohort size can lead to short-run and/or long-run financial instability of the pension system. Irregular developments of fertility are, however, only one reason why a NDC pension system might not be capable of achieving a balanced budget, neither in the short nor in the long run. There exist a large number of other factors that might change in an erratic fashion, e.g. sudden changes in the average fertility age, in the average age of labor market entry, in the age-earnings profiles or in age-specific mortality. It is an interesting area for future research to study and systematize the effects of these changes and to analyze their interaction with increasing life expectancy.

Given that there are many sources for unpredictable shocks it seems inevitable that a NDC system includes some additional mechanism that adjusts for unforeseen imbalances like the Swedish "automatic balance mechanism" (Settergren 2012, Auerbach & Lee 2009). Independent of the design of such an additional balance mechanism it is important to note, however, that the appropriate definition of the notional interest rate and remaining life expectancy will in any case lead to a more stable system and will make the activation of the automatic balance mechanism a less frequent event.

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Appendices

A Derivations and proofs

A.1 Derivations for section 3.1

Using the expressions (11), (12) and (13) in (7) one can write the expenditures of the NDC pension system as:

$$O(t) = \overline{\tau} \int_{\widetilde{R}(t)}^{\widetilde{T}(t)} \frac{\int_0^{R(t-a)} \left[W(t) e^{-\int_{t-a+b}^{t} g(s)\,\mathrm{d}s} e^{\int_{t-a+b}^{t-a+R(t-a)} \rho(s)\,\mathrm{d}s} \right] \mathrm{d}b}{\Gamma(t-a, R(t-a))} e^{\int_{t-a+R(t-a)}^{t} \vartheta(s)\,\mathrm{d}s} N(t-a)\,\mathrm{d}a,$$

where I have used the fact that $W(t-a+b) = W(t) e^{-\int_{t-a+b}^{t} g(s)\,\mathrm{d}s}$. For the case where $\vartheta(t) = \rho(t)$ one can use the fact that:

$$e^{\int_{t-a+b}^{t-a+R(t-a)} \rho(s)\,\mathrm{d}s} e^{\int_{t-a+R(t-a)}^{t} \rho(s)\,\mathrm{d}s} = e^{\int_{t-a+b}^{t} \rho(s)\,\mathrm{d}s}$$

to write $O(t)$ in the form as shown in (14).

A.2 Derivations for a proportional retirement age

It is assumed here that retirement age follows (21a) as in section 4.1.

A.2.1 Adjusted wage bill growth and period life expectancy

When the notional interest rate is set according to (17c) and one uses period life expectancy (16a) the following relation holds:

$$\int_0^{R(t-a)} e^{\int_{t-a+b}^{t} (\rho(s) - g(s))\,\mathrm{d}s}\,\mathrm{d}b = R(t-a) \tag{25}$$

and expression (14) simplifies to:

$$O(t) = \overline{\tau} W(t) \overline{N} \int_{\widetilde{R}(t)}^{\widetilde{T}(t)} \frac{R(t-a)}{\widetilde{T}(t-a+R(t-a)) - R(t-a)}\,\mathrm{d}a.$$

Using $R(t) = \mu T(t)$ and $\widetilde{T}(t) = \frac{T(t)}{1+\gamma}$ one can calculate that $\frac{R(t-a)}{\widetilde{T}(t-a+R(t-a))-R(t-a)} = \frac{\mu(1+\gamma)}{1-\mu}$ and thus $O(t) = \overline{\tau}W(t)\overline{N}\left(\widetilde{T}(t) - \widetilde{R}(t)\right)\frac{\mu(1+\gamma)}{1-\mu} = \overline{\tau}W(t)\overline{N}\widetilde{R}(t)$. Given that the revenues of the pension system are $I(t) = \overline{\tau}W(t)L(t) = \overline{\tau}W(t)\overline{N}\widetilde{R}(t)$ it follows that $D(t) = 0$ for every period. The NDC system is in this case permanently balanced.

A.2.2 Unadjusted wage bill growth and period life expectancy

Using (17b) and (16a) leads to the following relations (for (14)):

$$\int_{t-a+b}^{t} (\rho(s) - g(s))\,\mathrm{d}s = \int_{t-a+b}^{t} \frac{\gamma}{T(s)}\,\mathrm{d}s = \ln\left(\frac{T(t)}{T(t-a+b)}\right),$$

$$\int_0^{R(t-a)} e^{\int_{t-a+b}^{t}(\rho(s)-g(s))\,\mathrm{d}s}\,\mathrm{d}b = \frac{T(t)}{\gamma}\ln\left(1+\mu\gamma\right),$$

$$\int_{\widetilde{R}(t)}^{\widetilde{T}(t)} \frac{\frac{T(t)}{\gamma}\ln\left(1+\mu\gamma\right)}{\widetilde{T}(t-a+R(t-a)) - R(t-a)}\,\mathrm{d}a = T(t)\frac{(1+\gamma)\left(\frac{1+\gamma}{1+\mu\gamma}\right)\ln(1+\mu\gamma)}{\gamma^2(1-\mu)}.$$

The deficit ratio is therefore given as:

$$d(t) = \frac{(1+\gamma)(1+\mu\gamma)\ln\left(\frac{1+\gamma}{1+\mu\gamma}\right)\ln(1+\mu\gamma)}{\gamma^2\mu(1-\mu)} \approx 1 + \frac{\gamma}{2},$$

where the approximation follows from a first-order Taylor expansion around $\gamma = 0$. The use of unadjusted wage bill growth thus leads to a permanent deficit. This has also been confirmed by numerical simulations without using the approximation.

A.2.3 Adjusted wage bill growth and cohort life expectancy

This combines (17c) and (16b). Similar steps as in A.2.1 lead to $\frac{R(t-a)}{T(t-a)-R(t-a)} = \frac{\mu T(t)}{(1+\gamma)(1+\mu\gamma)}$ and thus $d(t) = \frac{1}{1+\gamma}$.

A.2.4 Unadjusted wage bill growth and cohort life expectancy

Combining (16b) and (17b) and using similar steps as in A.2.2 leads to:

$$d(t) = \frac{(1+\mu\gamma)\ln\left(\frac{1+\gamma}{1+\mu\gamma}\right)\ln(1+\mu\gamma)}{\gamma^2\mu(1-\mu)} \approx 1 - \frac{\gamma}{2}, \tag{26}$$

In this case one faces a permanent surplus as has also been confirmed by numerical simulations.

A.3 Derivations for constant retirement age

For this case it is assumed that retirement age is constant as specified in equation (21b) in section 4.2.

A.3.1 Adjusted wage bill growth and period life expectancy

When the notional interest rate is set according to (17c) and one uses period life expectancy (16a) the following relations hold:

$$\int_{t-a+b}^{t} (\rho(s) - g(s))\,\mathrm{d}s = \int_{t-a+b}^{t} -\frac{\gamma}{T(s)}\,\mathrm{d}s = \ln\left(\frac{T(t-a+b)}{T(t)}\right),$$

$$\int_0^{\overline{R}} e^{\int_{t-a+b}^{t} -\frac{\gamma}{T(s)}\,\mathrm{d}s}\,\mathrm{d}b = \frac{\overline{R}}{T(t)}\left(T(t-a) + \frac{\gamma}{2}\overline{R}\right),$$

$$\begin{aligned}
\int_{\overline{R}}^{\widetilde{T}(t)} \frac{\frac{\overline{R}}{T(t)}\left(T(t-a)+\frac{\gamma}{2}\overline{R}\right)}{\widetilde{T}(t-a+\overline{R})-\overline{R}}\,\mathrm{d}a &= \frac{\overline{R}(1+\gamma)}{T(t)} \int_{\overline{R}}^{\widetilde{T}(t)} \frac{T(t-a)+\frac{\gamma}{2}\overline{R}}{T(t-a)-\overline{R}}\,\mathrm{d}a \\
&= \frac{\overline{R}(1+\gamma)}{T(t)}\left(\overline{R}\left(\frac{2+\gamma}{2\gamma}\ln(1+\gamma)\right) + \frac{T(t)}{1+\gamma} - \overline{R}\right).
\end{aligned}$$

Since the total revenues of the system are given by $I(t) = \hat{\tau}W(t)\overline{N}\bar{R}$ the deficit ratio boils down to:

$$d(t) = \frac{\overline{R}(1+\gamma)}{T(t)}\left(\frac{(2+\gamma)\ln(1+\gamma)}{2\gamma} - 1\right) + 1, \tag{27}$$

which, using a first-order Taylor expansion around $\gamma = 0$, is approximately 1.

A.3.2 Adjusted wage bill growth and cohort life expectancy

Combining (16b) and (17c) and using similar steps as before one gets that:

$$d(t) = \frac{\overline{R}}{T(t)} \left(\frac{(2+\gamma)\ln(1+\gamma)}{2\gamma} - 1 \right) + \frac{1}{1+\gamma} \approx 1 - \gamma. \tag{28}$$

A.3.3 Average wage growth or wage bill growth and period or cohort life expectancy

For a notional interest rate according to (17c) or (17a) one can define a "hybrid" life expectancy concept that uses a mixture of period and cohort life expectancy, i.e. $\Gamma(t, R(t)) = \left[\eta T(t) + (1-\eta)(\widetilde{T}(t + R(t)))\right] - R(t)$, where η gives the relative weight of cohort life expectancy. It holds that:

$$\int_{t-a+b}^{t} 0 \, \mathrm{d}s = 0,$$

$$\int_{0}^{\overline{R}} e^{\int_{t-a+b}^{t} 0 \, \mathrm{d}s} \, \mathrm{d}b = \overline{R},$$

$$\begin{aligned} \int_{\overline{R}}^{\widetilde{T}(t)} \frac{\overline{R}}{(\eta T(t-a) + (1-\eta)(\widetilde{T}(t-a+\overline{R}))) - \overline{R}} \, \mathrm{d}a &= \int_{\overline{R}}^{\widetilde{T}(t)} \frac{\overline{R}(1+\gamma)}{(1+\eta\gamma)(T(t-a) - \overline{R})} \, \mathrm{d}a \\ &= \frac{\overline{R}(1+\gamma)}{1+\eta\gamma} \frac{\ln(1+\gamma)}{\gamma}. \end{aligned}$$

The deficits ratios for the cases with cohort and period life expectancy follow for $\eta = 1$ and $\eta = 0$, respectively.

One can calculate the "optimal" weight η^* that leads to a permanently balanced budget with $d(t) = 1, \forall t$. It comes out as:

$$\eta^* = \frac{1}{\gamma^2} \left((1+\gamma)\ln(1+\gamma) - \gamma \right) \approx \frac{1}{2} \left(1 - \frac{\gamma}{3} \right). \tag{29}$$

A mixture of both life expectancy concepts with $\eta = \eta^*$ will thus lead to a balanced NDC system in the case where the retirement age does not react to increasing life expectancy. For $\gamma = 1/4$, e.g., the optimal value is $\eta^* = 0.46$ and thus puts slightly more weight on period life expectancy.

A.4 Derivations for an optimal retirement age

The derivation of the optimal retirement age is not straightforward. The main difficulty stems from the fact that the optimal choice, given by the first-order condition (24), depends on $\frac{\partial \Omega(t)}{\partial R(t)}$, the reaction of lifetime income to prolonging the work life. This effect itself depends on the streams of pension payments which again depend — via the notional interest rate — on the retirement behavior of past and future cohorts. The way I deal with this problem is to first make a conjecture concerning the solution for $R(t)$. I will then use this solution to determine the path of notional interest rates and finally verify that — given this path — the conjectured choice of retirement is in fact the optimal solution.

A.4.1 The case where $V(T(t), a)$ is homogeneous of degree 0

My conjecture in this case is that the optimal retirement age is proportional to life expectancy, i.e. $R(t) = \mu T(t)$. From section 4.1 it is known that in this case $g^L(t) = \frac{\gamma}{T(t)}$. I start with the benchmark case where the notional interest rate is given by the adjusted growth rate of the wage bill (17c) and the calculation of remaining life expectancy is based on period life expectancy (16a). Under the assumption that wages are constant (i.e. $W(t+a) = \overline{W}$) it follows that $P(t, a) = \frac{\overline{\tau}\,\overline{W} R(t)}{\widetilde{R}(t+R(t)) - R(t)}$ and lifetime income can be written as:

$$\Omega(t) = (1 - \overline{\tau})\overline{W} R(t) + (T(t) - R(t)) \frac{\overline{\tau}\,\overline{W} R(t)}{\widetilde{R}(t + R(t)) - R(t)},$$

which has to be equal to lifetime consumption given by $\overline{C}T(t)$.

For $R(t) = \mu T(t)$ lifetime income reduces to $\Omega(t) = (1 + \overline{\tau}\,\overline{W}\gamma)R(t)$. On the other hand, the disutility of labor at the time of retirement can be written as $V(T(t), R(t)) = V(1, \frac{R(t)}{T(t)}) = V(1, \mu)$ which is a time-independent expression. Furthermore, $\frac{\partial \Omega(t)}{\partial R(t)} = \overline{W}(1 + \overline{\tau}\gamma)$ and $U'(\overline{C}) = \frac{1}{\overline{W}\mu(1+\overline{\tau}\gamma)}$. From (24) the solution for the optimal μ^* is thus implicitely given by the equation:

$$V(1, \mu^*) = \frac{1}{\mu^*}. \tag{30}$$

It can thus be concluded that $R(t) = \mu^* T(t)$ is in fact the optimal solution to the first-order condition (24) as conjectured in the first place. If $V(T(t), a) = \upsilon$ (as used by Kalemli-Ozcan & Weil (2010)), i.e. disutility of labor is constant and age-independent, the solution to (30) is given by $\mu^* = \frac{1}{\upsilon}$. If $V(T(t), a) = \upsilon e^{\frac{a}{T(t)}}$ (as used by Bloom et al. (2007)) the solution can be derived (numerically) from $\frac{1}{\mu^*} = \upsilon e^{\mu^*}$.

In order to derive the optimal solutions for other combinations of the assumptions concerning the notional interest rate and remaining life expectancy one has to repeat these steps. It can be shown that under the conjecture that $R(t) = \mu T(t)$ each combination of assumptions (17a) to (17c) and assumptions (16a) and (16b) leads to a situation where a generation's total pension income can be approximated (around $\gamma = 0$) as: $\int_{R(t)}^{T(t)} P(t,a)\,\mathrm{d}a = \chi R(t)\overline{\tau}\overline{W}$, for some parameter χ. Lifetime income is thus approximately given as: $\Omega(t) = R(t)\overline{W}\left(1+\overline{\tau}(\chi-1)\right)$ and the constant consumption level is given by: $\overline{C}(t) = \frac{R(t)\overline{W}(1+\overline{\tau}(\chi-1))}{T(t)}$. From these expressions one can derive that $\frac{\partial\Omega(t)}{\partial R(t)}U'(\overline{C}(t)) = \frac{T(t)}{R(t)} = \frac{1}{\mu}$. This is the same result as for the case with assumptions (17c) and (16a) and again confirms the conjecture that $R(t) = \mu T(t)$.

A.4.2 The case where $V(T(t), a) = \upsilon a$

For this case the conjecture is that the optimal retirement age is given by $R(t) = \sqrt{\frac{T(t)}{\upsilon}}$. The period-specific retirement age $\widetilde{R}(t)$ can be derived as: $\widetilde{R}(t) = \frac{-\gamma+\sqrt{\gamma^2+4\upsilon T(t)}}{2\upsilon}$ and thus the growth rate of the labor force comes out as: $g^L(t) = \frac{\gamma}{2T(t)}\left(1+\frac{\gamma}{\sqrt{\gamma^2+4\upsilon T(t)}}\right)$. Similarly as in the case above, one can show that for all combinations of assumptions concerning the notional interest rate and remaining life expectancy the total pension income can be approximated as:

$$\int_{R(t)}^{T(t)} P(t,a)\,\mathrm{d}a = \int_{R(t)}^{T(t)} \frac{\int_0^{R(t)} e^{\int_{t+b}^{t+a}\rho(s)\,\mathrm{d}s}\,\mathrm{d}b}{\Gamma(t,R(t)) - R(t)}\,\mathrm{d}a \approx \chi R(t).$$

Then one can again write $\frac{\partial\Omega(t)}{\partial R(t)}U'(\overline{C}(t)) = \frac{T(t)}{R(t)}$ and thus the first-order condition (24) reduces to $\upsilon R(t) = \frac{T(t)}{R(t)}$ which has the solution $R(t) = \sqrt{\frac{T(t)}{\upsilon}}$, thereby verifying the conjecture.

As in the cases with proportional and constant retirement age (cf. Tables 1 and 2) only the combination of assumptions (17c) and (16a) lead to a balanced budget. This is documented in Table 3.

Table 3: The deficit-ratio for a non-proportional retirement age

	(1)	(2)	(3)
	Notional Interest Rate — Growth Rate of:		
	Average Wages	Wage Bill	Adjusted Wage Bill
Period Life Expectancy	$\approx 1 + \frac{\gamma}{4}$	$\approx 1 + \frac{\gamma}{2}$	≈ 1
Cohort Life Expectancy	$\approx 1 - \frac{3\gamma}{4}$	$\approx 1 - \frac{\gamma}{2}$	$\approx 1 - \gamma$

Note: The table shows the period deficit ratio $d(t) = \frac{O(t)}{I(t)}$ for various assumption about the notional interest rate and the measure of remaining life expectancy. Period [cohort] life expectancy is defined as in (16a) [(16b)]. The notional interest rate is given by one of the expressions in (17a), (17b) and (17c), respectively. Furthermore, it is assumed that retirement age is given by $R(t) = \sqrt{\frac{T(t)}{v}}$.

B Simulations

In order to study the behavior of the NDC system for arbitrary patterns of retirement one has to rely on numerical simulations. For this purpose one also has to use a discrete-time version of the continuous-time set-up presented in section 3.1 of the paper. While allowing for a wide range of assumption, this discrete-time framework has the disadvantage that many of the precise results derived in the paper are only valid in an approximate sense. The main problem is that the discrete-time version only allows for integer values of life expectancy and retirement age while the development of life expectancy and the assumptions about parallel retirement adjustments involve non-integer values. The following equations are used for the simulation:

$$T_t = \text{Round}\left(T_0 + \gamma{\cdot}t\right),$$

$$L_t = \sum_{a=1}^{\widetilde{T}_t} I_{t-a+1,a} N_{t-a+1},$$

where $\widetilde{T}_t$ is the maximum age observed in period t and $I_{t,s}$ is an indicator variable with $I_{t,s} = 1$ if generation t works at age s and $I_{t,s} = 0$ otherwise.

$$B_t = \sum_{a=1}^{\widetilde{T}_t} (1 - I_{t-a+1,a}) N_{t-a+1},$$

$$z_t = \frac{B_t}{L_t},$$

$$I_t = \overline{\tau} \sum_{a=1}^{\widetilde{T}_t} I_{t-a+1,a} W_t N_{t-a+1},$$

$$O_t = \sum_{a=1}^{\widetilde{T}_t} (1 - I_{t-a+1,a}) P_{t-a+1,a} N_{t-a+1},$$

$$K_{t,R_t} = \sum_{a=1}^{R_t} \overline{\tau} W_{t+a-1} \Psi_{t+a-1,t+R_t},$$

where $\Psi_{t+a-1,t+R_t}$ is the cumulative notional interest rate factor given by:

$$\Psi_{t+a-1,t+R_t} = \prod_{s=t+a}^{t+R_t} (1 + \rho_s),$$

for $1 \leq a \leq R_t$ and ρ_t is the notional interest rate from period $t-1$ to t. The first pension that is received by generation t in period $t + R_t$ is given by:

$$P_{t,R_t} = \frac{K_{t,R_t}}{\Gamma_{t,R_t}}.$$

Existing pensions are adjusted according to:

$$P_{t,a} = P_{t,R_t} \Theta_{t+R_t,t+a-1},$$

where $\Theta_{t+R_t,t+a-1}$ is the cumulative adjustment rate factor given by:

$$\Theta_{t+R_t,t+a-1} = \prod_{s=t+R_t+1}^{t+a-1} (1 + \vartheta_s),$$

for $R_t + 2 \leq a \leq T_t$ and ϑ_t is the adjustment rate from period $t-1$ to t. The two possibilities for specifying the remaining life expectancy (equations (16a) and (16b)) are now given by:

$$\Gamma_{t,R_t} = \widetilde{T}_{t+R_t} - R_t,$$

$$\Gamma_{t,R_t} = T_t - R_t.$$

The different types of notional interest rate are again given by:

$$\rho_t = g_t^W,$$

$$\rho_t = g_t^W + g_t^L,$$

$$\rho_t = g_t^W + g_t^L - \frac{\gamma}{T_{t-1}},$$

where now $g_t^W = \frac{W_t - W_{t-1}}{W_{t-1}}$, $g_t^L = \frac{L_t - L_{t-1}}{L_{t-1}}$ and where $g_t^L = \frac{\gamma}{T_{t-1}}$ if the retirement age is chosen according to $\widetilde{R}_t = \mu \widetilde{T}_t$ (or the respective integer values thereof).